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Short Consideration for Application Examples of Health-Care System Integration Using Hyperfuzzy Sets and HyperRough Sets

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Abstract

To model diverse real-world phenomena, a range of uncertainty-handling concepts has been actively studied, including Fuzzy Sets, Rough Sets, Intuitionistic Fuzzy Sets, Paraconsistent Sets, Neutrosophic Sets, Hyperneutrosophic Sets, Plithogenic Sets, and others. Among these extensions of fuzzy sets, *Hyperfuzzy Sets* are of particular significance. A hyperfuzzy set extends the notion of fuzzy sets to a hierarchical structure, enabling a more refined and flexible representation of uncertainty. In addition, the notion of a *HyperRough Set* generalizes the classical setting to multi-attribute data by assigning, to each attribute profile, a subset of the universe and then taking rough approximations with respect to a fixed indiscernibility relation. However, research on real-life applications of Hyperfuzzy Sets and HyperRough Sets remains limited. This paper explores application examples drawn from real-world scenarios by examining system integration using the Hyperfuzzy Set and HyperRough Set frameworks. Note that *Health-Care system integration* is the process of connecting distinct subsystems or components into a unified, functional, and efficient whole.

Keywords: Hyperfuzzy set, Fuzzy set, System integration, Set theory, Rough set, Hyperrough set.

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1| Introduction

1.1| Fuzzy, Hyperfuzzy, and SuperHyperFuzzy Sets

A fuzzy set on a universe U is specified by a membership function $\mu_A : U \to [0, 1]$, which replaces crisp, binary inclusion with graded degrees of belonging and thus captures uncertainty at finer resolution [1, 2]. Numerous variants enrich this paradigm, including Bipolar Fuzzy Sets [3, 4], Intuitionistic Fuzzy Sets [5, 6], Neutrosophic Sets [7, 8, 9], Hesitant Fuzzy Sets [10, 11], Plithogenic Sets [12, 13, 14], and Picture Fuzzy Sets [15, 16, 17].

Intuitively, a *hyperfuzzy set* lifts the fuzzy idea to a layered (or hierarchical) representation, allowing membership information to be structured across levels; in this sense it strictly generalizes the classical fuzzy framework [18, 19, 20, 21, 22, 23]. More generally, an (m, n)-*superhyperfuzzy set* associates to each m-level subset an n-level family of membership descriptors, providing a calculus for recursively nested uncertainty and multi-scale complexity [24, 25, 26].

1.2| Rough and HyperRough Sets

Rough set theory provides a framework for handling uncertainty by approximating a target set using its *lower* and *upper* approximations defined with respect to an indiscernibility (equivalence) relation on the universe [27, 28]. Related extensions of Rough Sets include the Fuzzy Rough Set [29, 30, 31, 32], the Neutrosophic Rough Set[33, 34, 35], and the Weighted Rough Set[36, 37, 38], each introducing additional layers of uncertainty representation and flexibility.

A *HyperRough Set* generalizes the classical rough set model to multi-attribute contexts: for each attribute profile, one first assigns a subset of the universe and then constructs the corresponding lower and upper approximations under a fixed indiscernibility relation [39, 40, 41, 42, 43].

1.3| Our Contributions

The foregoing makes clear that advancing the theory and practice of Fuzzy, Hyperfuzzy, and SuperHyperFuzzy Sets, as well as Rough and HyperRough Sets, is of substantial importance. However, especially for Hyperfuzzy/SuperHyperFuzzy and HyperRough models, comprehensive application studies remain limited. To help close this gap, the present paper develops illustrative, real-world use cases focused on *system integration* within the Hyperfuzzy and HyperRough frameworks. For clarity, by *health-care system integration* we mean the process of connecting distinct subsystems or components into a unified, functional, and efficient whole.

2| Preliminaries

This section provides an introduction to the foundational concepts and definitions required for the discussions in this paper. It should be noted that only finite sets are considered throughout this paper.

2.1| Hyperfuzzy Set

A fuzzy set assigns to each element a membership degree in $[0, 1]$, thereby capturing uncertainty through more granular membership levels rather than a strict binary classification [1, 2]. Intuitively, a hyperfuzzy set extends the concept of fuzzy sets into a hierarchical structure, allowing for a more refined and flexible representation of uncertainty. In this sense, a hyperfuzzy set generalizes the traditional fuzzy set framework [18, 19, 20]. The formal definition is given below.

Definition 2.1 (Universal Set)**.** A *universal set*, denoted by U, is the set that contains all elements under consideration in a particular context. Every set discussed is assumed to be a subset of U.

Definition 2.2 (Powerset)**.** (see [44, 45, 46]) For a set S, the *powerset* $\mathcal{P}(S)$ is the collection of all subsets of S:

$$\mathcal{P}(S) = \{ A \subseteq S \}.$$

In particular, both the empty set $\emptyset$ and S itself lie in $\mathcal{P}(S)$.

Definition 2.3 (Fuzzy Set)**.** [1] A *fuzzy set* τ in a non-empty universe Y is a mapping $\tau : Y \to [0,1]$. A *fuzzy relation* on Y is a fuzzy subset δ in $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, then δ is called a *fuzzy relation on* τ if

$$\delta(y,z) \le \min\{\tau(y), \tau(z)\} \quad \text{for all } y, z \in Y.$$

Definition 2.4 (Hyperfuzzy Set)**.** [47, 48] Let X be a nonempty set. A mapping

$$\tilde{\mu} : X \to \tilde{P}([0,1])$$

is called a *hyperfuzzy set* over X, where $\tilde{P}([0,1])$ denotes the family of all nonempty subsets of the interval $[0,1]$.

Example 2.5 (Hyperfuzzy Set in Health Care: Sepsis-risk assessment under multiple scorers)**.** Let $X = \{\mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3\}$ be three inpatients evaluated for early sepsis. Three independent scorers are used: SOFA (s_1), qSOFA (s_2), and an ML model (s_3). A *hyperfuzzy set* $\tilde{\mu} : X \to \tilde{\mathcal{P}}([0,1])$ assigns to each patient the *set* of admissible membership degrees (risk levels) reported by the scorers:

$$\tilde{\mu}(\mathrm{p}_1) = \{0.72,\ 0.69,\ 0.80\}, \quad \tilde{\mu}(\mathrm{p}_2) = \{0.31,\ 0.40\}, \quad \tilde{\mu}(\mathrm{p}_3) = \{0.55,\ 0.61,\ 0.58\}.$$

Two useful summaries are $\min \tilde{\mu}(\mathrm{p}_1) = 0.69$, $\max \tilde{\mu}(\mathrm{p}_1) = 0.80$ (similarly for $\mathrm{p}_2, \mathrm{p}_3$), which bound per-patient risk while retaining the full hyperfuzzy information as sets of degrees.

2.2| (m, n)-superhyperfuzzy set

An (m, n)-superhyperfuzzy set maps each m-level subset to a family of n-level membership sets, modeling hierarchical recursive uncertainty and complexity [24, 25, 26].

Definition 2.6 (n-th powerset)**.** [49, 50, 51, 52, 53] For a set X, define $\mathcal{P}_1(X) = \mathcal{P}(X)$ and, for $n \ge 1$,

$$\mathcal{P}_{n+1}(X) \;=\; \mathcal{P}\big(\mathcal{P}_n(X)\big).$$

When excluding the empty set, write $\mathcal{P}_n^*(X) = \mathcal{P}_n(X) \setminus \{\emptyset\}$.

Example 2.7 (A concrete instance of the n-th powerset)**.** Let $X = \{a, b\}$. Then the first powerset is

$$\mathcal{P}_1(X) = \mathcal{P}(X) = \{\ \emptyset,\ \{a\},\ \{b\},\ \{a,b\}\ \},$$

so $|\mathcal{P}_1(X)| = 2^{|X|} = 4$.

The second powerset collects *subsets of* $\mathcal{P}_1(X)$:

$$\mathcal{P}_2(X) = \mathcal{P}\big(\mathcal{P}_1(X)\big),$$

which has $2^4 = 16$ elements. Typical elements of $\mathcal{P}_2(X)$ (i.e., subsets of $\mathcal{P}_1(X)$) are

$$A_1 = \{\{a\}\}, \qquad A_2 = \{\{b\}, \{a,b\}\}, \qquad A_3 = \mathcal{P}_1(X) \text{ (the full set)}, \qquad A_4 = \emptyset.$$

Thus $A_i \in \mathcal{P}_2(X)$ for $i = 1, 2, 3, 4$.

The third powerset is again a powerset, now of $\mathcal{P}_2(X)$:

$$\mathcal{P}_3(X) = \mathcal{P}\big(\mathcal{P}_2(X)\big),$$

so $|\mathcal{P}_3(X)| = 2^{16}$. An element of $\mathcal{P}_3(X)$ is a *set of elements of* $\mathcal{P}_2(X)$; for instance,

$$B = \{\ A_1, A_2\ \} \;\in\; \mathcal{P}_3(X).$$

If the empty set is excluded at each stage, the nonempty variants are $\mathcal{P}_1^*(X) = \{\{a\}, \{b\}, \{a,b\}\}$ and $\mathcal{P}_2^*(X) = \mathcal{P}_2(X) \setminus \{\emptyset\}$, etc. In particular, $A_4 = \emptyset \notin \mathcal{P}_2^*(X)$ but $A_1, A_2, A_3 \in \mathcal{P}_2^*(X)$. This illustrates the progression:

$$X \xrightarrow{\ \mathcal{P}\ } \mathcal{P}_1(X) \xrightarrow{\ \mathcal{P}\ } \mathcal{P}_2(X) \xrightarrow{\ \mathcal{P}\ } \mathcal{P}_3(X),$$

where each level consists of sets built from the objects of the previous level.

Definition 2.8 (m, n-SuperHyperFuzzy Set)**.** (cf.[43, 54]) Let X be a nonempty set and let $m, n \in \mathbb{N}_0$. Define the nonempty k-th powerset of a set Y by

$$\mathcal{P}_0^*(Y) = Y, \quad \mathcal{P}_k^*(Y) = \mathcal{P}\big(\mathcal{P}_{k-1}^*(Y)\big) \setminus \{\emptyset\}, \quad k \geq 1.$$

In particular, $\mathcal{P}_m^*(X)$ is the family of all nonempty elements of the m-th iterated powerset of X, and $\mathcal{P}_n^*([0,1])$ is defined analogously. Then an (m, n)-SuperHyperFuzzy Set on X is a function

$$\tilde{\mu}_{m,n} : \mathcal{P}_m^*(X) \longrightarrow \tilde{\mathcal{P}}_n^*\big([0,1]\big), \quad A \mapsto \tilde{\mu}_{m,n}(A),$$

where $\tilde{\mathcal{P}}_n^*([0,1])$ denotes the collection of all nonempty subsets of $\mathcal{P}_n([0,1])$. Thus each $A \in \mathcal{P}_m^*(X)$ is assigned a nonempty family of membership-degree sets $\tilde{\mu}_{m,n}(A) \subseteq \mathcal{P}_n([0,1])$, capturing hierarchical uncertainty across both the m- and n-levels.

Example 2.9 (SuperHyperFuzzy Set in Health Care: Discharge readiness for patient groups)**.** Let X be the current census on a ward and consider nonempty patient groups $A \in \mathcal{P}_1^*(X)$. Define an $(m, n) = (1, 1)$-*SuperHyperFuzzy* map

$$\tilde{\mu}_{1,1} : \ \mathcal{P}_1^*(X) \longrightarrow \tilde{\mathcal{P}}_1^*([0,1]),$$

where $\tilde{\mu}_{1,1}(A)$ is a *family* of membership-degree sets contributed by distinct policies. For instance, for $A = \{\mathrm{p}_1, \mathrm{p}_2\}$ ("step-down candidates"):

$$\tilde{\mu}_{1,1}(A) = \big\{\, [0.60, 0.80] \text{ (clinical guideline)}, \ \{0.55, 0.70\} \text{ (bed-management heuristic)} \,\big\}.$$

Thus one group-level input produces a set *of* admissible degree-sets, letting clinicians reconcile multiple governance sources while keeping their structure explicit.

2.3| Rough Sets and HyperRough Sets

Rough set theory offers a principled method for treating uncertainty by approximating a target set through its *lower* and *upper* envelopes with respect to an indiscernibility (equivalence) relation on the universe [27, 28].

Definition 2.10 (Rough Approximation)**.** [55, 56] Let X be a finite universe and let $R \subseteq X \times X$ be an equivalence relation. Denote the R-equivalence class of $x \in X$ by $[x]_R$. For $Y \subseteq X$, define the *R-lower* and *R-upper* approximations by

$$\mathrm{low}_R(Y) \ := \ \{\, x \in X : \ [x]_R \subseteq Y \,\}, \qquad \mathrm{up}_R(Y) \ := \ \{\, x \in X : \ [x]_R \cap Y \neq \varnothing \,\}.$$

Then $\mathrm{low}_R(Y) \subseteq Y \subseteq \mathrm{up}_R(Y)$. It is convenient to write the *boundary region* $B_R(Y) := \mathrm{up}_R(Y) \setminus \mathrm{low}_R(Y)$.

Example 2.11 (Rough Set in Health Care: Isolation decision under indiscernibility)**.** Let $X = \{x_1, \ldots, x_6\}$ be patients in an ED. Define an indiscernibility relation R by shared (age band, symptom cluster):

$$C_1 = \{x_1, x_2\}, \quad C_2 = \{x_3, x_4\}, \quad C_3 = \{x_5, x_6\} \quad (R\text{-equivalence classes}).$$

Let $Y = \{$"requires respiratory isolation"$\}$ be the target, with $Y = \{x_1, x_3, x_6\}$. Then the R-lower and R-upper approximations are

$$\mathrm{low}_R(Y) = \bigcup\{C_i : \ C_i \subseteq Y\} = \varnothing, \qquad \mathrm{up}_R(Y) = \bigcup\{C_i : \ C_i \cap Y \neq \varnothing\} = C_1 \cup C_2 \cup C_3 = X,$$

so the boundary $B_R(Y) = X \setminus \varnothing = X$ (decision is uncertain in every class). If, instead, $Z = \{x_1, x_2, x_3, x_4\}$, then

$$\mathrm{low}_R(Z) = C_1 \cup C_2 = \{x_1, x_2, x_3, x_4\}, \quad \mathrm{up}_R(Z) = C_1 \cup C_2 = \{x_1, x_2, x_3, x_4\}, \quad B_R(Z) = \varnothing,$$

exhibiting a case with *no* ambiguity.

The notion of a *HyperRough Set* generalizes the classical setting to multi-attribute data by assigning, to each attribute profile, a subset of the universe and then taking rough approximations relative to a fixed indiscernibility relation [43].

Definition 2.12 (HyperRough Set)**.** [42, 43] Let X be a nonempty finite universe and let $T_1, \ldots, T_n$ be attributes with respective domains $J_1, \ldots, J_n$. Put

$$J \ := \ J_1 \times \cdots \times J_n.$$

Fix an equivalence relation $R \subseteq X \times X$ (indiscernibility on X). A *HyperRough Set* on X is a pair (F, J) where

$$F : \ J \longrightarrow \mathcal{P}(X)$$

assigns to each attribute vector $a = (a_1, \ldots, a_n) \in J$ a subset $F(a) \subseteq X$. For each $a \in J$, the rough approximations of $F(a)$ are

$$\mathrm{low}_R\big(F(a)\big) \ = \ \{\, x \in X : \, [x]_R \subseteq F(a) \,\}, \qquad \mathrm{up}_R\big(F(a)\big) \ = \ \{\, x \in X : \, [x]_R \cap F(a) \neq \varnothing \,\}.$$

These satisfy, for all $a \in J$,

$$\mathrm{low}_R\big(F(a)\big) \ \subseteq \ \mathrm{up}_R\big(F(a)\big), \qquad \mathrm{low}_R(\varnothing) = \mathrm{up}_R(\varnothing) = \varnothing, \qquad \mathrm{low}_R(X) = \mathrm{up}_R(X) = X,$$

and are monotone in the argument: if $F(a) \subseteq F(b)$ then $\mathrm{low}_R(F(a)) \subseteq \mathrm{low}_R(F(b))$ and $\mathrm{up}_R(F(a)) \subseteq \mathrm{up}_R(F(b))$.

Example 2.13 (HyperRough Set in Health Care: High-risk triage across attribute profiles)**.** Let $X = \{x_1, \ldots, x_5\}$ be admitted patients. Attributes are $T_1 = \mathrm{Fever} \in \{\mathrm{yes}, \mathrm{no}\}$ and $T_2 = \mathrm{Comorbidity} \in \{\mathrm{yes}, \mathrm{no}\}$, so $J = T_1 \times T_2 = \{(\mathrm{y}, \mathrm{y}), (\mathrm{y}, \mathrm{n}), (\mathrm{n}, \mathrm{y}), (\mathrm{n}, \mathrm{n})\}$. Define $F : J \to \mathcal{P}(X)$, where $F(a)$ collects patients deemed *high risk* under profile a:

$$F(\mathrm{y}, \mathrm{y}) = \{x_1, x_3\}, \quad F(\mathrm{y}, \mathrm{n}) = \{x_2\}, \quad F(\mathrm{n}, \mathrm{y}) = \{x_4\}, \quad F(\mathrm{n}, \mathrm{n}) = \varnothing.$$

Let R be indiscernibility by ward: $C_1 = \{x_1, x_2\}$, $C_2 = \{x_3, x_4, x_5\}$. For $a = (\mathrm{y}, \mathrm{y})$ we have $F(a) = \{x_1, x_3\}$, hence

$$\mathrm{low}_R\big(F(a)\big) = \varnothing \quad (\text{no } C_i \text{ contained in } F(a)), \qquad \mathrm{up}_R\big(F(a)\big) = C_1 \cup C_2 = \{x_1, x_2, x_3, x_4, x_5\}.$$

If we refine F to $F'(\mathrm{y}, \mathrm{y}) = C_2 = \{x_3, x_4, x_5\}$, then $\mathrm{low}_R(F'(\mathrm{y}, \mathrm{y})) = \mathrm{up}_R(F'(\mathrm{y}, \mathrm{y})) = C_2$, illustrating how HyperRough sets encode profile-wise targets $F(a)$ while rough approximations are taken uniformly with respect to the fixed R.

3| Review and Results: Example of Application Examples of System Integration Using Hyperfuzzy Sets

System Integration connects diverse subsystems or components into a unified system, ensuring seamless functionality, interoperability, efficiency, and coordinated operations [57, 58, 59]. Here, we present example application cases of system integration using Hyperfuzzy Sets.

Example 3.1 (Smart Building Energy Management)**.** Consider integrating three subsystems in a smart building:

$$U = \{\mathrm{HVAC}, \ \mathrm{Lighting}, \ \mathrm{Security}\}.$$

Define a Hyperfuzzy set of *energy-efficiency demands*

$$H_{\mathrm{energy}} = \big\{ \, (\, u, \ \tilde{\mu}_{\mathrm{energy}}(u) \,) \mid u \in U \big\},$$

where

$$\tilde{\mu}_{\mathrm{energy}} : U \to \widetilde{\mathcal{P}}([0,1])$$

is a fuzzy-set-valued membership:

$$\tilde{\mu}_{\mathrm{energy}}(\mathrm{HVAC}) = [0.7, 0.9], \quad \tilde{\mu}_{\mathrm{energy}}(\mathrm{Lighting}) = [0.5, 0.8], \quad \tilde{\mu}_{\mathrm{energy}}(\mathrm{Security}) = [0.3, 0.5].$$

System integration proceeds by computing the hyperfuzzy intersection of energy demands with occupancy patterns, yielding adaptive control rules.

Example 3.2 (Industrial IoT Fault Detection)**.** In an Industrial IoT network, let

$$U = \{\mathrm{Sensor}_1, \ \mathrm{Sensor}_2, \ \mathrm{Actuator}\}.$$

Define the Hyperfuzzy fault-likelihood set

$$H_{\mathrm{fault}} = \big\{ \, \big(u, \ \tilde{\mu}_{\mathrm{fault}}(u)\big) \mid u \in U \big\},$$

with

$$\tilde{\mu}_{\text{fault}}(u) = [\,\underline{\mu}(u),\, \overline{\mu}(u)\,] \subseteq [0,1],$$

e.g. $\tilde{\mu}_{\text{fault}}(\text{Sensor}_1) = [0.2, 0.4]$, $\tilde{\mu}_{\text{fault}}(\text{Sensor}_2) = [0.6, 0.8]$, $\tilde{\mu}_{\text{fault}}(\text{Actuator}) = [0.1, 0.3]$. Integration uses hyperfuzzy aggregation to trigger maintenance alerts when combined likelihood exceeds a threshold interval.

Example 3.3 (Healthcare Information System)**.** Integrate Electronic Health Records, Monitoring Devices, and Billing:

$$U = \{\text{EHR},\ \text{Monitor},\ \text{Billing}\}.$$

Define a Hyperfuzzy trust-level set

$$H_{\text{trust}} = \big\{\, \big(u,\ \tilde{\mu}_{\text{trust}}(u)\big) \mid u \in U \big\},$$

where

$$\tilde{\mu}_{\text{trust}}(u) = [T_L(u),\, T_U(u)] \subseteq [0,1],$$

for instance $\tilde{\mu}_{\text{trust}}(\text{EHR}) = [0.8, 0.95]$, $\tilde{\mu}_{\text{trust}}(\text{Monitor}) = [0.6, 0.85]$, $\tilde{\mu}_{\text{trust}}(\text{Billing}) = [0.4, 0.7]$. System integration applies hyperfuzzy composition to ensure only data with sufficiently high trust intervals are shared across modules.

Example 3.4 (Smart Agriculture Irrigation Management)**.** Integrate three subsystems in a precision-farming setup:

$$U = \{\text{SoilSensor},\ \text{WeatherStation},\ \text{IrrigationController}\}.$$

Define the Hyperfuzzy *moisture-need set*

$$H_{\text{moisture}} = \big\{\, \big(u,\ \tilde{\mu}_{\text{moisture}}(u)\big) \mid u \in U \big\},$$

with

$$\tilde{\mu}_{\text{moisture}}(\text{SoilSensor}) = [0.2, 0.5], \quad \tilde{\mu}_{\text{moisture}}(\text{WeatherStation}) = [0.3, 0.6],$$
$$\tilde{\mu}_{\text{moisture}}(\text{IrrigationController}) = [0.4, 0.8].$$

Hyperfuzzy integration computes the intersection of soil and weather demands to drive adaptive irrigation schedules.

Example 3.5 (Supply Chain Risk Integration)**.** Consider three elements of a supply chain:

$$U = \{\text{SupplierReliability},\ \text{InventoryLevel},\ \text{TransportAvailability}\}.$$

Define the Hyperfuzzy *risk-assessment set*

$$H_{\text{risk}} = \big\{\, \big(u,\ \tilde{\mu}_{\text{risk}}(u)\big) \mid u \in U \big\},$$

where

$$\tilde{\mu}_{\text{risk}}(\text{SupplierReliability}) = [0.1, 0.3], \quad \tilde{\mu}_{\text{risk}}(\text{InventoryLevel}) = [0.4, 0.7],$$
$$\tilde{\mu}_{\text{risk}}(\text{TransportAvailability}) = [0.2, 0.5].$$

Hyperfuzzy aggregation yields a combined risk interval; thresholds trigger contingency integration protocols.

Example 3.6 (Autonomous Vehicle Sensor–Control Coordination)**.** Integrate perception and control modules:

$$U = \{\text{PerceptionModule},\ \text{LocalizationModule},\ \text{ControlModule}\}.$$

Define the Hyperfuzzy *confidence-level set*

$$H_{\text{confidence}} = \big\{\, \big(u,\ \tilde{\mu}_{\text{conf}}(u)\big) \mid u \in U \big\},$$

with example intervals

$$\tilde{\mu}_{\text{conf}}(\text{PerceptionModule}) = [0.6, 0.9], \quad \tilde{\mu}_{\text{conf}}(\text{LocalizationModule}) = [0.5, 0.8],$$
$$\tilde{\mu}_{\text{conf}}(\text{ControlModule}) = [0.7, 0.95].$$

System integration uses hyperfuzzy fusion to adjust control commands based on the weakest-link confidence interval.

Example 3.7 (Smart Grid Orchestration via $(m,n) = (2,1)$–SuperHyperfuzzy Set)**. Universe and levels.** Let the device universe be

$$X = \{\mathrm{PV}_A, \mathrm{BAT}_A, \mathrm{LOAD}_A, \mathrm{PV}_B, \mathrm{BAT}_B, \mathrm{LOAD}_B\}.$$

We model *orchestratable bundles of device–groups* during a heatwave as elements of $\mathcal{P}_2^*(X)$ (nonempty families of nonempty subsets of X). Consider

$$A = \{\, A_1,\ A_2 \,\} \in \mathcal{P}_2^*(X), \quad A_1 = \{\mathrm{PV}_A, \mathrm{BAT}_A\}, \quad A_2 = \{\mathrm{PV}_B, \mathrm{LOAD}_B\},$$

representing two coupled sub-bundles to be co-dispatched.

Membership as a family of degree sets. Choose $(m,n) = (2,1)$. Define a SuperHyperfuzzy membership $\tilde{\mu}_{2,1}(A) \in \tilde{\mathcal{P}}_1^*([0,1])$ (a nonempty family of subsets of $[0,1]$) encoding *dispatchability* from two estimators $(\mathsf{EST}_1, \mathsf{EST}_2)$:

$$\tilde{\mu}_{2,1}(A) = \Big\{ [0.68, 0.82],\ [0.74, 0.88] \Big\}.$$

SHF–AND (envelope aggregator). For a family $\mathcal{S} \subseteq \mathcal{P}_1([0,1])$, define

$$\mathsf{AND}_{\min}(\mathcal{S}) := \big[\min_{S\in\mathcal{S}} \inf S,\ \ \min_{S\in\mathcal{S}} \sup S \ \big].$$

Then the integrated readiness interval is

$$\tilde{\mu}_{\mathrm{ready}}(A) = \mathsf{AND}_{\min}\big(\tilde{\mu}_{2,1}(A)\big) = \big[\min(0.68, 0.74),\ \min(0.82, 0.88)\big] = [0.68,\ 0.82].$$

Action policy. Let thresholds $\tau_{\mathrm{auto}} = 0.75$ and $\tau_{\mathrm{review}} = 0.65$.

$$\begin{array}{ll} \text{Auto–orchestrate} & \text{if } L \geq \tau_{\mathrm{auto}}, \\ \text{Orchestrate w/ operator review} & \text{if } L < \tau_{\mathrm{auto}} \leq U \ \text{ and } \ U \geq \tau_{\mathrm{review}}, \\ \text{Defer} & \text{if } U < \tau_{\mathrm{review}}, \end{array}$$

where $[L, U] = [0.68, 0.82]$. Since $L = 0.68 < 0.75$ but $U = 0.82 \geq 0.75$,

Proceed with orchestration behind operator review.

Interpretation. Both estimators support feasibility up to 0.82, but the lower bound 0.68 (from EST_1) prevents fully automatic co-dispatch.

Example 3.8 (Warehouse Multi-Robot Task Allocation via $(m,n) = (1,2)$–SuperHyperfuzzy Set)**. Universe and levels.** Let $X = \{r_1, r_2, r_3, r_4, r_5\}$ be robots. Consider a team

$$A = \{r_1, r_2, r_4\} \in \mathcal{P}_1^*(X).$$

We choose $(m,n) = (1,2)$ so that membership collects *scenario-wise* families of degree-sets.

Membership as a family of *families* of degree sets. Define $\tilde{\mu}_{1,2}(A) \in \tilde{\mathcal{P}}_2^*([0,1])$ with two scenarios:

$$\tilde{\mu}_{1,2}(A) = \Big\{ \ \Sigma_1 = \{[0.70, 0.85], [0.65, 0.80]\}, \quad \Sigma_2 = \{[0.55, 0.72]\} \Big\}.$$

Here Σ_1 = normal peak hours (two estimators), Σ_2 = degraded comms.

Two-stage SHF–AND. First compute an envelope per scenario Σ:

$$E(\Sigma) := \mathsf{AND}_{\min}(\Sigma) = \big[\min_{S\in\Sigma} \inf S,\ \min_{S\in\Sigma} \sup S\big].$$

Thus $E(\Sigma_1) = [0.65, 0.80]$, $E(\Sigma_2) = [0.55, 0.72]$. Adopt a robust (worst-case) aggregator across scenarios:

$$\mathsf{RobAND}\big(\tilde{\mu}_{1,2}(A)\big) := \big[\ \min_{\Sigma} \inf E(\Sigma),\ \min_{\Sigma} \sup E(\Sigma) \ \big] = [0.55,\ 0.72].$$

Action policy. With $\tau_{\mathrm{auto}} = 0.70$ and $\tau_{\mathrm{safe}} = 0.60$,

$$\begin{array}{ll} \text{Auto–allocate} & \text{if } L \geq \tau_{\mathrm{auto}}, \\ \text{Supervised allocate} & \text{if } \tau_{\mathrm{safe}} \leq L < \tau_{\mathrm{auto}} \text{ and } U \geq \tau_{\mathrm{auto}}, \\ \text{Defer/replan} & \text{if } L < \tau_{\mathrm{safe}}. \end{array}$$

For $[L, U] = [0.55, 0.72]$, $L < 0.60$, so

Defer allocation and replan under degraded communications.

Interpretation. The level-2 structure explicitly captures adverse scenarios, driving a conservative integration decision.

4| Review and Results: Example of Application Examples of System Integration Using HyperRough Sets

Here, we present example application cases of system integration using HyperRough Sets.

Example 4.1 (Smart Building Integration: HVAC–Lighting–Access)**.** Let the universe $X = \{z_1, \ldots, z_6\}$ denote six building zones. Define the indiscernibility relation R by *same floor*, with equivalence classes

$$[z_1]_R = \{z_1, z_2\}, \quad [z_3]_R = \{z_3, z_4\}, \quad [z_5]_R = \{z_5, z_6\}.$$

Let attributes be $\mathsf{Occ} \in \{\text{Low}, \text{High}\}$ (occupancy) and $\mathsf{Per} \in \{\text{Day}, \text{Night}\}$ (period), so $J = \{\text{Low}, \text{High}\} \times \{\text{Day}, \text{Night}\}$. For the profile $a = (\text{High}, \text{Night})$, suppose the integration target is the set of zones requiring *cooling + lighting boost*:

$$F(a) = \{z_1, z_2, z_4, z_5\} \subseteq X.$$

Then, using the rough approximations w.r.t. R,

$$\begin{aligned}
\text{low}_R\big(F(a)\big) &= \{x \in X : [x]_R \subseteq F(a)\} = \{z_1, z_2\}, \\
\text{up}_R\big(F(a)\big) &= \{x \in X : [x]_R \cap F(a) \neq \varnothing\} = \{z_1, z_2, z_3, z_4, z_5, z_6\} = X, \\
B_R\big(F(a)\big) &= \text{up}_R(F(a)) \setminus \text{low}_R(F(a)) = \{z_3, z_4, z_5, z_6\}.
\end{aligned}$$

Integration policy.

- Apply immediate HVAC/lighting actions to $\text{low}_R(F(a)) = \{z_1, z_2\}$ (definite need).
- For boundary zones $B_R(F(a))$, enable sensors and short-interval rechecks; escalate only if demand persists.
- Zones outside $\text{up}_R(F(a))$ receive no change (none in this instance).

This realizes a floor-consistent control strategy: decisive actions occur only when an entire floor-class warrants them, while ambiguous floors are handled conservatively.

Example 4.2 (Hospital IT Integration: EHR–Monitors–Lab Systems)**.** Let $X = \{r_1, \ldots, r_8\}$ denote record fragments across three systems (EHR, bedside Monitor, Lab). Define R by *same patient ID*, yielding

$$[r_1]_R = \{r_1, r_2\}, \quad [r_3]_R = \{r_3, r_4\}, \quad [r_5]_R = \{r_5, r_6\}, \quad [r_7]_R = \{r_7, r_8\}.$$

Let attributes be patient $\mathsf{Consent} \in \{\text{No}, \text{Yes}\}$ and clinical $\mathsf{Criticality} \in \{\text{Low}, \text{High}\}$; thus $J = \{\text{No}, \text{Yes}\} \times \{\text{Low}, \text{High}\}$. For $a = (\text{Yes}, \text{High})$, take the target set of records eligible for *cross-module sharing with alerts*:

$$F(a) = \{r_1, r_2, r_3, r_6, r_7\}.$$

Then

$$\begin{aligned}
\text{low}_R\big(F(a)\big) &= \{x : [x]_R \subseteq F(a)\} = \{r_1, r_2\} \quad \text{(only the full class } \{r_1, r_2\} \subseteq F(a)), \\
\text{up}_R\big(F(a)\big) &= \{x : [x]_R \cap F(a) \neq \varnothing\} = \{r_1, \ldots, r_8\} = X, \\
B_R\big(F(a)\big) &= X \setminus \{r_1, r_2\} = \{r_3, r_4, r_5, r_6, r_7, r_8\}.
\end{aligned}$$

Integration policy.

- Auto-sync and notify across EHR, Monitor, Lab for $\text{low}_R(F(a))$ (patient-level completeness guaranteed).
- For the boundary $B_R(F(a))$, gate sharing behind consent/role checks and clinician override; queue reconciliation jobs to complete missing counterparts.
- Deny propagation outside $\text{up}_R(F(a))$ (none here).

Thus, only patients with *complete* consented high-criticality bundles are integrated automatically; partial bundles are handled safely.

Example 4.3 (Supply-Chain Integration: WMS–TMS–ERP Coordination). Let $X = \{s_1, \dots, s_9\}$ represent shipments. Define R by *same route group*, with

$$[s_1]_R = \{s_1, s_2, s_3\}, \quad [s_4]_R = \{s_4, s_5, s_6\}, \quad [s_7]_R = \{s_7, s_8, s_9\}.$$

Attributes: $\mathsf{Weather} \in \{\text{Mild}, \text{Severe}\}$, $\mathsf{Urgency} \in \{\text{Low}, \text{High}\}$; hence $J = \{\text{Mild}, \text{Severe}\} \times \{\text{Low}, \text{High}\}$. For $a = (\text{Severe}, \text{High})$, let the target be shipments to *reroute or expedite*:

$$F(a) = \{s_1, s_2, s_3, s_6, s_7\}.$$

Then

$$\begin{aligned} \text{low}_R\big(F(a)\big) &= \{x : [x]_R \subseteq F(a)\} = \{s_1, s_2, s_3\} \quad (\text{entire route group } \{s_1, s_2, s_3\} \subseteq F(a)), \\ \text{up}_R\big(F(a)\big) &= \{x : [x]_R \cap F(a) \neq \varnothing\} = \{s_1, \dots, s_9\} = X, \\ B_R\big(F(a)\big) &= X \setminus \{s_1, s_2, s_3\} = \{s_4, s_5, s_6, s_7, s_8, s_9\}. \end{aligned}$$

Integration policy.

- WMS–TMS–ERP immediately co-execute rerouting and capacity reservation for $\text{low}_R(F(a))$ (route-consistent action).
- For $B_R(F(a))$, request carrier/weather confirmation and inventory checks; commit actions only if the remaining route members flip into $F(a)$.
- No actions for items outside $\text{up}_R(F(a))$ (none here).

This enforces route-coherent interventions while avoiding fragmented decisions on partially affected routes.

5| Review and Results: Example of Application Examples of HealthCare-System Integration Using Hyperfuzzy Sets

A healthcare system is an integrated network that delivers services, coordinates data, finances care, and optimizes population health outcomes effectively[60, 61, 62, 63]. This section explains HealthCare-System Integration Using Hyperfuzzy Sets.

Example 5.1 (HealthCare-System Integration Using Hyperfuzzy Sets — Sepsis Early-Warning Pipeline). **Subsystems and universe.** Let

$$U = \{\text{EHR}, \ \text{BedsideMonitor}, \ \text{Lab}\}.$$

Define a hyperfuzzy set of *trust* for a real-time sepsis early-warning pipeline:

$$H_{\text{trust}} = \{(u, \ \tilde{\mu}_{\text{trust}}(u)) \mid u \in U\}, \qquad \tilde{\mu}_{\text{trust}}(u) \subseteq [0, 1] \text{ (interval)}.$$

For a particular ICU context, set the interval memberships (higher = more trustworthy):

$$\tilde{\mu}_{\text{trust}}(\text{EHR}) = [0.85, \ 0.95], \quad \tilde{\mu}_{\text{trust}}(\text{BedsideMonitor}) = [0.70, \ 0.90], \quad \tilde{\mu}_{\text{trust}}(\text{Lab}) = [0.60, \ 0.80].$$

Integration operator (hyperfuzzy AND). Using the Gödel t-norm (min) lifted to intervals,

$$\bigwedge\nolimits_G \big\{[l_i, u_i]\big\}_{i=1}^{n} := \big[\min_i l_i, \ \min_i u_i\big].$$

Hence the pipeline *readiness* interval is

$$\tilde{\mu}_{\text{ready}} = \bigwedge\nolimits_G \big\{[0.85, 0.95], \ [0.70, 0.90], \ [0.60, 0.80]\big\} = \big[\min(0.85, 0.70, 0.60), \ \min(0.95, 0.90, 0.80)\big] = [0.60, \ 0.80].$$

Action policy (explicit thresholds). Let

$$\tau_{\text{auto}} = 0.75, \qquad \tau_{\text{review}} = 0.65.$$

Decision rule:

$$\begin{array}{ll} \text{Auto-integrate} & \text{if } L \geq \tau_{\text{auto}}, \\ \text{Integrate with human review} & \text{if } L < \tau_{\text{auto}} \leq U \text{ and } U \geq \tau_{\text{review}}, \\ \text{Defer/block} & \text{if } U < \tau_{\text{review}}. \end{array}$$

With $[L, U] = [0.60, 0.80]$, we have $L = 0.60 < 0.75$ but $U = 0.80 \geq 0.75$, so the second clause applies:

$$\text{Proceed with integration behind clinician review (alert gated).}$$

All three subsystems jointly support the pipeline ($U = 0.80$), but the weakest link (Lab lower bound 0.60) prevents fully automatic action; the hyperfuzzy interval captures this uncertainty explicitly.

Example 5.2 (HealthCare-System Integration Using Hyperfuzzy Sets — Remote Diabetes Titration)**. Subsystems and universe.** Let

$$U = \{\text{EHRMedicationList},\ \text{PharmacyDispense},\ \text{WearableAdherence}\}.$$

Define a hyperfuzzy *data-reliability* set

$$H_{\text{reli}} = \{(u,\ \tilde{\mu}_{\text{reli}}(u)) \mid u \in U\}, \qquad \tilde{\mu}_{\text{reli}}(u) = [l(u), u(u)] \subseteq [0, 1].$$

For a telehealth visit:

$$\tilde{\mu}_{\text{reli}}(\text{EHRMedicationList}) = [0.90,\ 0.98],$$
$$\tilde{\mu}_{\text{reli}}(\text{PharmacyDispense}) = [0.92,\ 0.99],$$
$$\tilde{\mu}_{\text{reli}}(\text{WearableAdherence}) = [0.85,\ 0.95].$$

Integration operator (product t-norm on intervals). Using the product t-norm $T_\Pi(x_1, \ldots, x_n) = \prod_i x_i$ lifted to intervals,

$$\bigwedge\nolimits_\Pi \{[l_i, u_i]\}_{i=1}^{n} := \Big[\ \prod_i l_i,\ \prod_i u_i\ \Big].$$

Thus the *titration-readiness* interval equals

$$\begin{aligned}\tilde{\mu}_{\text{titrate}} &= \bigwedge\nolimits_\Pi \Big\{[0.90, 0.98],\ [0.92, 0.99],\ [0.85, 0.95]\Big\}\\ &= \Big[\ 0.90 \cdot 0.92 \cdot 0.85,\ \ 0.98 \cdot 0.99 \cdot 0.95\ \Big].\end{aligned}$$

Explicitly,

$$L = 0.90 \times 0.92 \times 0.85 = 0.702, \qquad U = 0.98 \times 0.99 \times 0.95 = 0.92169,$$

so

$$\tilde{\mu}_{\text{titrate}} = [0.702,\ 0.92169].$$

Action policy (dose-change gating). Let the auto-titration threshold be $\tau_{\text{auto}} = 0.70$ and the safety floor be $\tau_{\text{safe}} = 0.60$.

$$\begin{cases}\text{Auto-titrate} & \text{if } L \geq \tau_{\text{auto}},\\ \text{Issue recommendation + confirm} & \text{if } L < \tau_{\text{auto}} \leq U \text{ and } U \geq \tau_{\text{safe}},\\ \text{Do not titrate} & \text{if } U < \tau_{\text{safe}}.\end{cases}$$

Since $L = 0.702 \geq 0.70$, we obtain

$$\text{Auto-titrate is enabled (high-confidence integration across systems).}$$

High reliability across EHR, pharmacy, and adherence yields a tight, high readiness interval; the product t-norm enforces that all subsystems must be strong to pass the automatic threshold.

Example 5.3 (Perioperative Scheduling Integration via $(m, n) = (2, 1)$)**. Universe and levels.** Let

$$X = \{\text{OR1},\ \text{AnesthesiaTeam},\ \text{SterileProc},\ \text{PACU}\}.$$

For a specific case, coordinatable *bundles of unit-groups* are in $\mathcal{P}_2^*(X)$. Consider

$$A = \big\{\ \{\text{OR1}, \text{SterileProc}\},\ \{\text{AnesthesiaTeam}, \text{PACU}\}\ \big\}.$$

Membership as readiness families. Choose $(m, n) = (2, 1)$. Let $\tilde{\mu}_{2,1}(A)$ collect checklists and feeds (room turnover, tray status, staff availability):

$$\tilde{\mu}_{2,1}(A) = \Big\{\ [0.80, 0.95],\ [0.70, 0.90],\ [0.60, 0.85]\ \Big\}.$$

Integration and policy. Envelope:

$$\tilde{\mu}_{\text{ready}}(A) = \mathrm{AND}_{\min}\big(\tilde{\mu}_{2,1}(A)\big) = [\min(0.80, 0.70, 0.60),\ \min(0.95, 0.90, 0.85)] = [0.60,\ 0.85].$$

With thresholds $\tau_{\text{book}} = 0.75$, $\tau_{\text{hold}} = 0.65$:

$$\begin{array}{ll} \text{Auto–book} & \text{if } L \geq 0.75, \\ \text{Book w/ charge–nurse confirm} & \text{if } L < 0.75 \leq U \text{ and } U \geq 0.65, \\ \text{Hold} & \text{if } U < 0.65. \end{array}$$

Here $L = 0.60 < 0.65$ but $U = 0.85 \geq 0.75 \Rightarrow$

Book behind charge–nurse confirmation; expedite tray turnaround.

The weakest readiness feed (sterilization) pulls the lower bound down, preventing an automatic commit.

Example 5.4 (Multimodal Cancer Diagnosis Integration via $(m, n) = (1, 2)$)**. Universe and levels.** Let diagnostic subsystems be

$$X = \{\text{Radiology},\ \text{Pathology},\ \text{Genomics}\}.$$

For a tumor board candidate, consider $A = \{\text{Radiology}, \text{Pathology}\} \in \mathcal{P}_1^*(X)$. Choose $(m, n) = (1, 2)$ so membership aggregates *scenario families*.

Membership (scenario families). Define $\tilde{\mu}_{1,2}(A)$ with two scenarios:

$$\tilde{\mu}_{1,2}(A) = \Big\{\ \Sigma_1 = \{[0.80, 0.92], [0.75, 0.90]\},\quad \Sigma_2 = \{[0.58, 0.72]\}\Big\},$$

where Σ_1 = complete imaging + preliminary histology; Σ_2 = pending stains / discordant read.

Two-stage envelope and decision. Per-scenario envelopes: $E(\Sigma_1) = [0.75, 0.90]$, $E(\Sigma_2) = [0.58, 0.72]$. Robust aggregator:

$$\tilde{\mu}_{\text{integrate}}(A) = \mathrm{RobAND}\big(\tilde{\mu}_{1,2}(A)\big) = [\min(0.75, 0.58),\ \min(0.90, 0.72)] = [0.58,\ 0.72].$$

Policy with $\tau_{\text{auto}} = 0.70$, $\tau_{\text{review}} = 0.60$:

Queue for tumor board with radiology–pathology consensus review (not auto-escalated).

While one scenario is strong, the level-2 family captures a plausible weaker path; robust integration mandates expert review.

6| Conclusion

This short paper explored application examples drawn from real-world scenarios by examining system integration using the Hyperfuzzy Set and HyperRough Set frameworks. In the future, we plan to pursue research on application cases of system integration within the settings of Neutrosophic Sets[64, 65, 66], Plithogenic Sets[14, 13], HyperSoft Sets[67, 12, 68], Graphs[69], HyperGraphs[70, 71], and SuperHyperGraphs[72, 73, 74].

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Data Availability

This manuscript presents purely conceptual work without empirical data. Scholars interested in these ideas are invited to undertake experimental or case-study research to substantiate and extend the proposed frameworks.

Ethical Approval

This paper involves no human or animal subjects and thus did not require ethics committee review or approval.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Conflicts of Interest

The authors declare that there are no competing interests concerning the content or publication of this article.

Disclaimer

The theoretical models and propositions herein have not yet been subjected to practical validation. Readers should independently verify all citations and be aware that inadvertent inaccuracies may remain. The opinions expressed are those of the authors and do not necessarily represent the views of affiliated organizations.

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